UNIFIED COUNCIL
Foundation for Success

UNIFIED COUNCIL OLYMPIADS

UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9

Question Paper Code : UM9257

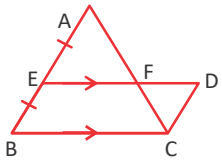
KEY

1	2	3	4	5	6	7	8	9	10
A	D	C	B	D	C	C	A	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	C	C	D	C	B	B	A	D	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	C	B	A	B	D	C	A	D	B
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A,B,C,D	B,D	A,B,C	B,D	A,C,D	B	D	C	A	B
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	A	D	C	C	B	B	A	D

EXPLANATIONS

MATHEMATICS - 1

1: (A) Area of parallelogram BCDE = Area of the trapezium BCFE + area of ΔCDF



= Area of the trapezium BCFE + area of ΔAEF [$\because \Delta CDF \cong \Delta AEF$]

= Area of ΔABC

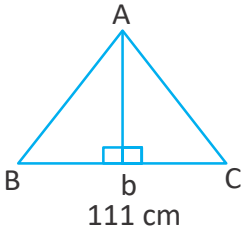
= 80 cm^2

2. (D) From options $x = 5$ then

$$\frac{\sqrt{5+4}+\sqrt{5-4}}{\sqrt{5+4}-\sqrt{5-4}} = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

3: (C) $s = \frac{a+b+c}{2} = \frac{105cm+36cm+111cm}{2}$

$= 126\,cm$



$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{126 \times 21 \times 90 \times 15}\ cm^2$

$= \sqrt{6 \times 21 \times 21 \times 6 \times 15 \times 15}\ cm^2$

$= 6 \times 21 \times 15\ cm^2$

$= 1890\ cm^2$

$\therefore \frac{1}{2} \times 111\ cm \times h = 1890\ cm^2$

$h = 1890\ cm^2 \times \frac{2}{111 cm}$

$= \frac{1260\ cm}{37}$

$= 34.05\ cm$

4: (B) Given both length are equal and perimeters are equal $\Rightarrow$ Adjacent side to length are also equal. But hypotenuse is greater than remaining two sides of a right angled triangle.

$\therefore$ Breadth of a rectangle > height of the parallelogram

$\therefore$ Area of the rectangle > Area of the parallelogram

5: (D) Given $2\pi r(h + r) = 1232\ cm^2$

$2 \times \frac{22}{7} \times r(21 + r) = 1232\ cm^2$

$21r + r^2 = 1232 \times \frac{7}{22} \times \frac{1}{2}$

$r^2 + 21r - 196 = 0$

$r^2 + 28r - 7r - 196 = 0$

$r(r + 28) - 7(r + 28) = 0$

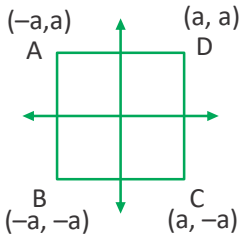
$(r + 28)(r - 7) = 0$

$r = 7$ (or) $r = -28$ is rejected

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 21\ cm^3$

$= 3234\ cm^3$

6: (C) It is a square



If the abscissas of A & B are same then distance between the points = difference of ordinates

$= a - (-a)$

side $= 2a$

$\therefore$ Area $= s^2 = (2a)^2 = 4a^2$

7: (C) Given $lb = A_1$ units^2, $bh = A_2$ units^2, $hl = A_3$ units^2

$\therefore lb \times bh \times hl = A_1A_2A_3$ units^6

$l^2b^2h^2 = A_1A_2A_3$ units^6

$lbh = \sqrt{A_1A_2A_3}$ units^3

$\therefore$ Volume $= \sqrt{A_1A_2A_3}$ units^3

8: (A) Given $\frac{x^2 + y^2}{xy} = 1$

$\therefore x^2 - xy + y^2 = 0$

$\therefore x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$= (x + y)(0)$

$= 0$

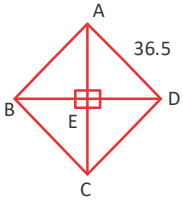
9: (B) Side of the rhombus $= \frac{146 cm}{4} = 36.5\ cm$

Given AC = 55 cm

$\therefore AE = \frac{AC}{2} = 27.5\ cm$

In ΔAED, $\angle AED = 90° \Rightarrow$

$ED^2 = AD^2 - AE^2$



$= (36.5)^2 - (27.5)^2$

$= (36.5 + 27.5)(36.5 - 27.5)$

$= 64 \times 9$

ED = $\sqrt{64 \times 9}$ = 8 × 3 = 24 cm

$\therefore$ BD = 2 × ED = 48 cm

Area of the rhombus = $\frac{1}{2}$ × AC × BD

$= \frac{1}{2} \times 55 \times 48$ cm^2

= 1320 cm^2

10: (A) $\frac{\sqrt[6]{12}}{\sqrt{3} \times \sqrt[3]{2}} = \frac{\sqrt[6]{12}}{\sqrt[6]{3^3} \times \sqrt[6]{2^2}} = \sqrt[6]{\frac{12}{27 \times 4}} = \sqrt[6]{\frac{1}{9}}$

$= \left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{6}} = \sqrt[3]{\frac{1}{3}}$

11: (C) Given f(x) = $(x + 1)^4$

$= ax^4 + bx^3 + cx^2 + dx + e$

$\therefore$ f(–1) = $(-1 + 1)^4$

$= a(-1)^4 + b(-1)^3 + c(-1)^2 + d(-1) + e$

$\therefore$ a – b + c – d + e = 0

$\therefore$ a + c + e = b + d

12. (C) $\sqrt{\frac{\left(\sqrt{4 \times 3} - \sqrt{4 \times 2}\right)\left(\sqrt{3} + \sqrt{2}\right)}{\left(5 + \sqrt{4 \times 6}\right)}}$

$= \sqrt{\frac{2\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right)}{\left(5 + 2\sqrt{6}\right)}}$

$= \sqrt{\frac{2(3-2)}{3+2+2\sqrt{3} \times \sqrt{2}}}$

$= \sqrt{\frac{2}{\left(\sqrt{3} + \sqrt{2}\right)^2}}$

$= \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{\left(\sqrt{3} - \sqrt{2}\right)}{\left(\sqrt{3} - \sqrt{2}\right)}$

$= \sqrt{6} - 2$

13: (C) Let $x = \sqrt{30 + \sqrt{30 + \sqrt{30 + ----}}}$

squaring on both sides

$x^2 = 30 + \sqrt{30 + \sqrt{30 + \sqrt{30 + ----}}}$

$\therefore x^2 = 30 + x$

$x^2 - x - 30 = 0$

$x^2 - 6x + 5x - 30 = 0$

$x = 6$ (or) $x = -5$

14: (D) $\left(x^{\frac{1}{3}} + x^{\frac{-1}{3}}\right)\left(x^{\frac{2}{3}} - 1 + x^{\frac{-2}{3}}\right)$

$= \left(x^{\frac{1}{3}} + x^{\frac{-1}{3}}\right)\left[\left(x^{\frac{1}{3}}\right)^2 - x^{\frac{1}{3}} \times x^{\frac{-1}{3}} + \left(x^{\frac{-1}{3}}\right)^2\right]$

$= \left(x^{\frac{1}{3}}\right)^3 + \left(x^{\frac{-1}{3}}\right)^3$

$= \left(x + x^{-1}\right)$

15: (C) Given $x + y + z = 21$

$\therefore x - 7 + y - 5 + z - 9 = 0$

Let a = $x - 7$, b = $y - 5$ & c = $z - 9$ then

a + b + c = 0 $\Rightarrow$ $a^3 + b^3 + c^3 = 3abc$

$= 3(x - 7)(y - 5)(z - 9)$

16: (B) Given $l = 3x$, b = $2x$ & h = x

Given total surface area = 88 cm^2

$\therefore$ 2($lb + bh + hl$) = 88 cm^2

$[(3x)(2x) + (2x)(x) + (x)(3x)] = \frac{88}{2}$cm^2

$6x^2 + 2x^2 + 3x^2 = 44$ cm^2

$11x^2 = 44$ cm^2

$x^2 = \frac{44}{11}$cm^2

$\therefore x = 2$

Volume = $lbh = (3x)(2x)(x) = 6x^3$

= 6 × 8 cm^3 = 48 cm^3

17: (B) Given $v_1 : v_2 = 2 : 3$ & $r_1 : r_2 = 1 : 2$

$$\therefore \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2 = 2 : 3$$

$$\Rightarrow \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{2}{3}$$

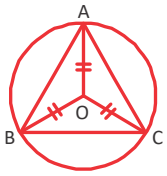
$$\left(\frac{1}{2}\right)^2 \times \frac{h_1}{h_2} = \frac{2}{3}$$

$$\frac{1}{4} \times \frac{h_1}{h_2} = \frac{2}{3}$$

$$\frac{h_1}{h_2} = \frac{8}{3}$$

$$h_1 : h_2 = 8 : 3$$

18: (A) 'O' is the circumcentre of ΔABC



$$\therefore \angle BAC = \frac{\angle BOC}{2} = \frac{96^\circ}{2} = 48^\circ$$

19: (D) It is in the form of

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$\therefore$ But $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$\therefore \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = (a + b + c)$$

$= 23 + 34 + 45 = 102$

20: (B) (0, 2), (–2, 1) and (1, 2.5)

lie on $x - 2y + 4 = 0$

21: (C) Option A all points lie on the line

$x + y = 0$

Option B all points lie on x - axis

Option D all points lie on the line

$y = x - 2$

Option C points form a quadrilateral

22: (C) Given $\angle A + \angle B + \angle C = 180^\circ$

$\angle B + 15^\circ + \angle C + 30^\circ + \angle C = 180^\circ$

$\angle C + 30^\circ + 15^\circ + 2\angle C + 30^\circ = 180^\circ$

$3\angle C = 105^\circ$

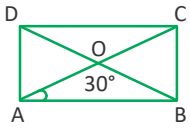
$\angle C = 35^\circ$

$\angle B = \angle C + 30^\circ = 65^\circ$

$\angle A = \angle B + 15^\circ = 80^\circ$

23: (B) In a triangle the third side is less than sum of two sides and greater than difference of the two sides.

24: (A) In ΔAOD, $\angle OAD = 90^\circ - 30^\circ = 60^\circ$



$\therefore$ AOD is an equilateral triangle

$\therefore$ AO = OD = AD = BC = 5 cm

25: (B) In ΔABC, $\angle B = 90^\circ \Rightarrow AC^2 = AB^2 + BC^2$

$AC = \sqrt{64 + 225} = 17$ cm

In ΔACD, $\angle ACD = 90^\circ \Rightarrow AD^2 = AC^2 + CD^2$

$145^2 = 17^2 + CD^2$

$CD^2 = 145^2 - 17^2 = (145 + 17)(145 - 17)$

$CD = \sqrt{162 \times 128}$

$= \sqrt{9 \times 9 \times 2 \times 2 \times 8 \times 8}$

$= 144$

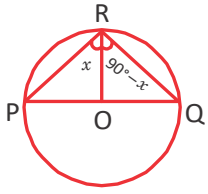
Area of the quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2} \times 8 \times 15 \text{ cm}^2 + \frac{1}{2} \times 17 \times 144 \text{ cm}^2$$

$= 60 \text{ cm}^2 + 1224 \text{ cm}^2$

$= 1284 \text{ cm}^2$

26: (D) <u>construction</u>:- Join OP & RQ



In ΔPQR, $\angle PRQ = 90^\circ$ [$\because$ Angle in a semicircle]

$\therefore \angle ORQ = 90° - x$

in ΔQOR, $90° - x + 90° - x + \angle ROQ$

$= 180°$

$\therefore \angle ROQ = 180° - 180° + 2x$

$= 2x$

27: (C) LHS $= \dfrac{(2x+3)-(x+2)-(x+1)}{(x+1)(x+2)(2x+3)}$

$= \dfrac{2x+3-x-2-x-1}{(x+1)(x+2)(2x+3)}$

$= \dfrac{2x+3-2x-3}{(x+1)(x+2)(2x+3)}$

$= 0$

28: (A) LHS $= \sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\sqrt{50}+\sqrt{72}$

$= \sqrt{2}+\sqrt{2\times2\times2}+\sqrt{3\times3\times2}+\sqrt{4\times4\times2}$
$+\sqrt{5\times5\times2}+\sqrt{6\times6\times2}$

$= \sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+5\sqrt{2}+6\sqrt{2}$

$= 21\sqrt{2} \qquad = \sqrt{441\times2}$

$= \sqrt{882}$

29: (D) $\sqrt{2x-7+2\sqrt{x^2-7x+12}}$

$= \sqrt{x-4+x-3+2\sqrt{(x-4)}\sqrt{(x-3)}}$

$= \sqrt{\left(\sqrt{x-4}\right)^2+\left(\sqrt{x-3}\right)^2+2\sqrt{x-4}\sqrt{x-3}}$

$= \sqrt{\left(\sqrt{x-4}+\sqrt{x-3}\right)^2}$

$= \left(\sqrt{x-4}+\sqrt{x-3}\right)$

30: (B) Given diameter of sphere

= side of cube = 2r

$\therefore$ Surface areas ratio of sphere and cube
$= 4\pi r^2 : 6(2r)^2$

$= 4 \times \dfrac{22}{7} \times r^2 : 6 \times 4 \times r^2$

$= 11 : 7 \times 3$

$= 11 : 21$

MATHEMATICS - 2

31: (A,B,C,D)

Let $(2x^2 + x)$ be a $\Rightarrow a^2 - 9a + 18$

$= a^2 - 6a - 3a + 18$

$= (a - 6)(a - 3)$

$= (2x^2 + x - 6)(2x^2 + x - 3)$

$= (2x^2 + 4x - 3x - 6)(2x^2 + 3x - 2x - 3)$

$= (2x - 3)(x + 2)(x - 1)(2x + 3)$

32: (B,D)

Given $f(t - 6) = 0$

$\Rightarrow (t - 6)^2 + t - 6 - 6 = 0$

$\Rightarrow t^2 - 12t + 36 + t - 12 = 0$

$\Rightarrow t^2 - 11t + 24 = 0$

$\Rightarrow t^2 - 8t - 3t + 24 = 0$

$t = 8$ (or) 3

33: (A, B, C)

$\sqrt{28+16\sqrt{3}} = \sqrt{4\times7+4\times4\sqrt{3}}$

$= 2\sqrt{7+4\sqrt{3}}$

$= 2\sqrt{7+2\sqrt{12}}$

$= 2\sqrt{4+3+2\sqrt{3}\times\sqrt{4}}$

$= 2\sqrt{2^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}}$

$= 2\sqrt{\left(2+\sqrt{3}\right)^2} \quad = 2\left(2+\sqrt{3}\right)$

$= 4+2\sqrt{3} = \left(\sqrt{3}+1\right)^2$

34: (B,D)

Given $\pi R^2 + \pi r^2 = 116\pi$

$\pi(R^2 + r^2) = 116\pi$

$R^2 + r^2 = 116$

But $(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

$(R + r)^2 + 6^2 = 2 \times 116$

$(R + r)^2 = 232 - 36$

$R + r = \sqrt{196} = 14$

$R + r = 14$ & $R - r = 6$

$\therefore R = 10$ & $r = 4$

35: (A, C, D)

Options A, C, D are true about a cyclic trapezium

REASONING

36. (B) Step-1 : Orange and white boxes interchange every time

Step-2 : For every time the shape rotate 90 degrees right

37. (D) (A) $15 + 5 - 8 + 4 - 6 < 6$

$24 - 14 < 6$

$10 < 6$ (False)

(B) $15 \div 15 + 8 < 4 + 6 \div 3$

$1 + 8 < 4 + 2$

$9 < 6$ (False)

(C) $15 > 5 \times 8 \div 4 + 6 \times 3$

$15 > 10 + 18$

$15 > 28$ (False)

(D) $15 \div 5 < 8 \div 4 + 6 \div 3$

$3 < 2 + 2$

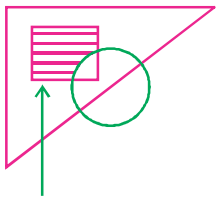
$3 < 4$ (True)

38. (C) 4 =  , 6 = 

39. (A) 

40. (B) All except small pox caused by bacteria, while small pox is caused by virus

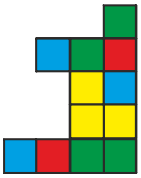
41. (A) In figure (X), the dot is placed in the region which is common to the triangle and square. Now, we have to search similar common region in four options. Only in option (a), we find such a region which is common to the triangle and square



Common to the triangle and square

42. (C) F and C are overlapped E rotate 90° left we get the shape W

Same relation is applied to the second image.

43. (A) 

44. (D) Statement 3 & 2 : purple red is written as Vet Get "

Statement 1 & 2 : "purple is written as "Get"

Therefore "red is coded as Vet"

Black orange yellow [Purple] blue →

set jet let [get] bet

(grey) green ◇red◇ [Purple] → [get]

(pet) wet ◇vet◇

[Purple] blue ◇red◇ silver → ◇vet◇

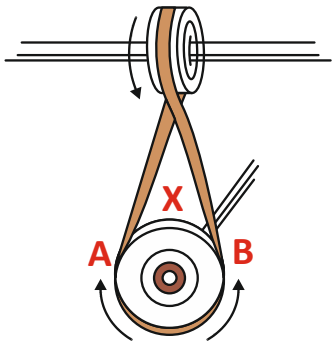
[get] tet (grey) orange pink → (pet)

45. (C) Daughter

My mother's only ground son's mother means → lady herself daughter of lady means the girl is the daughter of Lady.

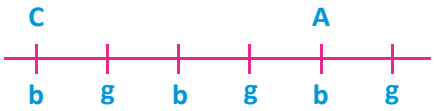
CRITICAL THINKING

46. (C) A



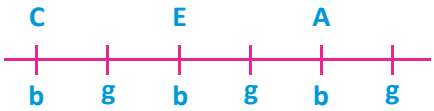
47. (B) According to the given data boys and girls are arranged in this way

In question it is given C is sitting Extreme left and A is sitting second seat from right end. so



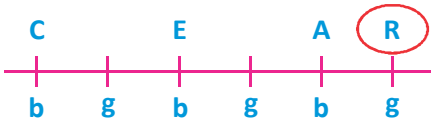
According to the statement (iii) and (v) Q is standing. So P, R, E and S are compulsory seated

E is boy sitting 3rd position



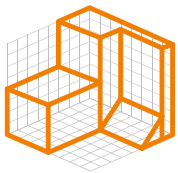
According to statement (iv) E and R not seated side by side

Therefore R is seated right end (immediate right of A)



48. (B) False

49. (A)



50. (D) According to the statement 1 Rahul is 25 years younger than his father

And statement 2 Rahul bother is 28 years younger than his father

So, Rahul's brother is 3 younger than Rahul

Hence , Rahul is born in 1999

Both statements required to the given question